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Monte Carlo Simulation for Reliability Estimation of Phased-Mission Systems With Minimum Operational Time Requirement

Xiaoyue Wu[11] Jane Hillston[2]
1. College of Information Systems and Management,
National University of Defense Technology,
Changsha, Hunan, 410073, China
2. School of Informatics,
The University of Edinburgh,
Edinburgh, Scotland, EH8 9AB, UK

ABSTRACT

This paper presents a Monte Carlo simulation approach for evaluating the mission reliability of a phased mission system (PMS) that has time redundancy in mission execution during some phases. This means that the system is not required to remain operational throughout the time interval of the phase; instead it just needs to work normally for a time interval no less than a given length. A Monte Carlo simulation procedure and the associated flowcharts are provided in detail. A simplified PMS is used as an illustrative example. Both the point estimate and the confidence interval of the mission reliability are derived by the proposed approach. The mission reliability is compared with that of PMS without time redundancy. The results show that the mission reliability may be seriously under-estimated if time redundancy is not taken into consideration.

KEYWORDS: mission reliability; reliability evaluation; phased-mission systems; time redundancy; Monte Carlo; simulation

NOMENCLATURE

PMS Phased-mission system
TT&C The spaceflight telemetry, tracking, and control system.

1 INTRODUCTION

For many engineering systems, particularly those designed to support critical missions, mission reliability is of paramount concern for system engineers. For example, each space flight mission requires reliable support by a spaceflight telemetry, tracking, and control (TT&C) system [1]. Phased-mission system (PMS) refers to a dynamic system that executes its mission in consecutive phases, and its configuration, operational requirements and the duration of the phase may change from phase to phase [2]. For example, a TT&C system is a typical PMS. Since a spacecraft orbits the earth, it can pass over a ground station for only a limited period of time and receives TT&C service based on predefined programs, which specify the detailed TT&C tasks of a facility during consecutive specific time intervals within its time window. Thus, a TT&C system can be regarded as a PMS with each of its task intervals as a phase.

Because of its theoretical and practical application importance, the reliability of PMS has attracted significant attention from researchers in the reliability community around the world. Xing and Amari gave an excellent survey of the research on mission reliability of PMS [2]. The existing approaches for reliability evaluation of PMS can generally be classified into the two categories: the analytical approach and the simulation approach [2] [3] [4]. The analytical approach uses various mathematical reliability models like Boolean algebraic models [5], fault tree analysis (FTA) models [6], binary decision diagram (BDD) models [7], continuous-time Markov chains (CTMC) [8],[9] and the combined methods [10]. It can provide precise estimates of the mission reliability. The simulation approach mainly uses Monte-Carlo methods, Petri nets [11]-[13] and various discrete event simulation approaches [14] to simulate the state transition process of system behavior and makes estimation by statistical inference based on simulation samples. In comparison with the analytical approach, the simulation approach is more flexible in modeling the dynamic behavior of the

[1] Corresponding Author: xiaoyuewucn@nudt.edu.cn
Tel: +86-731-84574553; Fax: +86-731-84573528.

system and has fewer restrictions on system characteristics under study, such as the distributions of the failure time and the repair time of system components.

As a PMS, TT&C system has some special features. Depending on the schedule and mission requirements, during some phases the system must operate continuously throughout the time, while for some other phases, only a minimum service support duration is required. Therefore, some of the phases may have time redundancy or flexibility in the starting time of mission execution, meaning that the mission can be executed from any time within the phase interval, provided that the system remains in the operational state continuously for a period of time not less than a given value. Such PMS can be called PMS with time redundancy.

To the best of our knowledge, the reliability of PMS with time redundancy has not been investigated previously. Wu studied the problem of mission reliability for systems with time redundancy in [15], and defined a type of mission reliability of systems within an interval of time with a minimum operational time requirement. However, the approach is applicable only to one-unit systems and for a single phase mission. Later, Wu and Hillston [16] studied the mission reliability of semi-Markov systems with time redundancy under the assumption that the system can have multiple repairable components, whose failure and repair times can have semi-Markov distributions. However, the study is limited to semi-Markov systems with only one phase.

This paper presents a general Monte Carlo simulation approach for estimating the mission reliability of PMS with time redundancy in some phases. For such kind of systems, different phases can have different mission success criteria (for instance, one phase requires minimum continuously operational time interval, while another phase requires the system to remain operational throughout the phase time). Thus, the feature of time redundancy can be taken into consideration in mission reliability evaluation of PMS to make the results more close to practical values.

This rest of the paper is organized as follows: In Section 2, the main procedure for Monte-Carlo simulation of PMS is introduced; In Section 3, an example PMS is given and its mission reliability is estimated by the proposed simulation procedure. The simulation results are provided and analyzed. Section 6 gives some conclusions and describes possible future work.

2 SIMULATION PROCEDURES

2.1 Main Assumptions

In this paper, we make the following assumptions:

(1) The system and all of its components can only be in either an up or a down state. At the beginning of the first phase, all the components are in the up states.

(2) When a component enters a down state, the repair work begins immediately. The component is "as good as new" after the repair work is done.

(3) Different components are s-independent. There is no statistical dependence between their failures or repairs. Either the failure time or the repair time can follow any type of distributions (not need to be restricted to exponential distribution).

(4) The overall mission of the system succeeds if and only if the mission is successful in all phases. There are two types of criteria for mission success in phases. For the first type, the mission will succeed in the phase if the system remains operational throughout the whole duration of the phase, while for the second type, the mission will succeed if the system remains operational for a minimum given time length within the duration of the phase.

(5) A component can be inactive in a given phase, where it can only be repaired, and no failure occurs.

(6) The duration of each phase is deterministic.

2.1 Main Simulation Procedure

To evaluate the mission reliability of PMS with time redundancy, we use a Monte Carlo simulation procedure. Considering the s-independence of all the components in each phase, we can sample the failure and repair durations for each component during all the mission times independently. Then, we can find all the time points at which the state of the system changes, and check by the structure functions of the corresponding phase to find whether the system is in the up or the down state space at these time points. From these results, the set of time spans for which the system in the up and the down states can be built, so whether the mission succeeds or fails in each phase can be judged based on the mission success criterion of the phase. As a result, using a large enough number of simulation runs, the mission reliability of the PMS can be estimated.

In Fig. 1, we describe the procedure of the Monte-Carlo simulation approach for mission reliability estimation. The main steps are as follows.

Step 1: Initialization. Set the number of simulation runs to be $simN$.

Step 2: Iterate to collect results of all simulation runs. The procedure for each run will be given in the following subsection. If the mission succeeds in the k th run, let $Iss_k = 1$; otherwise $Iss_k = 0$.

Step 3: The point estimation of the mission reliability is given by

$$\hat{R} = \frac{1}{simN} \sum_{k=1}^{simN} Iss_k \qquad (1)$$

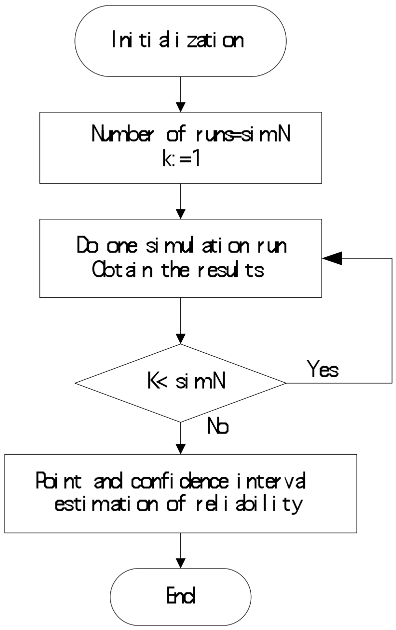

Figure 1 The Main Simulation Procedure

In addition to getting a point estimation of the mission reliability, we give the confidence interval of the mission reliability by using the Wald method [17]. The $100(1-\alpha)\%$ confidence interval is given by

$$\hat{R} \pm z_{\alpha/2}\sqrt{\frac{\hat{R}(1-\hat{R})}{simN}} \qquad (2)$$

where $z_{\alpha/2}$ is the percentile of the standard normal distribution [17].

2.2 Procedure for One Simulation Run

As shown in Fig. 1, we need the results of all simulation runs for estimation of the mission reliability of the PMS. Fig.2 shows the main procedure for obtaining the results of one simulation run.

The main steps are explained as follows:

Step 1: Iterate for all components in the system to generate a sample for each component [18][19]. Detailed procedure will be introduced later in this section.

Step 2: Produce a merged sample set of components.

We call each period during which a component remains in a state a span; thus each span is operational or down. To judge the success of the mission, it is necessary to combine the samples of all components obtained previously in order to build a scenario of the evolution of the system states during the mission time in the following way.

(1) Scan the previously obtained samples of all components; create a list to record the time points when there is change of any component state, and the states of spans corresponding to these points.

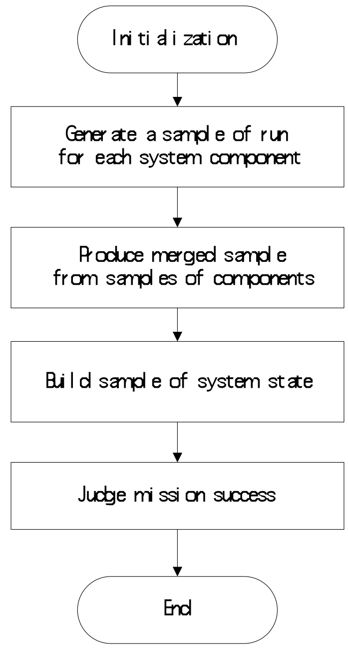

Figure 2 Flowchart of One Simulation Run

(2) Reorder time points and spans in the list according to their time precedence to form a new totally ordered list of time points for all components.

(3) Trim the resulting list according to the mission time. To do this, we need to insert the phase starting and ending times in the previous time point list, and delete those spans that exceed the mission time.

Step 3: Build a sample of the system state.

(1) Scan time points of state change in the previous list, for each one, get its phase and the states of all components involved in that phase.

(2) Based on the structure function of the corresponding phase, determine the (macro) state of the system by the states of components involved in the phase.

(3) Merge all adjacent operational spans of system (since there may exist some consecutive spans that are of operational or non-operational state in the previous list, so we need to combine them).

Step 4: Judge whether the mission is successful for this run based on the results of mission success in each phase.

$$I_{sys} \leftarrow \prod_{i=1}^{M} Is_i \qquad (3)$$

where, $Is_i \leftarrow 1$ if the mission succeeds in phase i; otherwise, $Is_i \leftarrow 0$;

If $I_{sys} = 1$, then the mission succeeds; otherwise, the mission fails.

2.3 Generating sample for component

In the previous subsections, we relied upon obtaining samples of state evolution for each component.

For convenience, we use the following symbols in the simulation procedure.

cp : current component

cT : current simulation clock time

f_{Prev} : state of the preceding sampled span of cp, it equals 1 if the preceding span is of up state, and equals 0 if the preceding span is of down state.

$remT$: remaining life time or remaining repair time of cp. This variable is necessary for simulation in case of nonexponential distributions, which do not have memoryless property.

The sampling procedure can be described as follows.

First, set initial values for related variables. Set cT as the start time of the phase for which cp first enters into use in the mission. Let $f_{Prev} = 0$, $remT = 0$.

Then, perform the following steps repeatedly until cT reaches mission time T_m.

Step 1: Set values for variables of current phase. Let Ph be the phase to which cT belongs. Set f_{Lt} equal 1 if Ph is an inactive phase for cp, otherwise, set it as 0. Let Te be the end time of Ph.

Step 2: Obtain a sampled span according to whether Ph is an active phase for cp.

In the sequel, we use $dueT$ to denote the next possible state change time of the component; use wT to denote the next simulation clock time.

(1) Case 1: Ph is an active phase ($f_{Lt} = 0$).

If $remT > 0$, then let $dueT := cT + remT$;

If $remT = 0$, then do as follows:

If $f_{\Pr ev} = 0$, meaning the previous span was in the down state, so sample an operational span Lup according to the failure time distribution of cp, and let $waitT := Lup$.

If $f_{\Pr ev} = 1$, meaning the previous span was in down state, so sample a repair time span $Ldown$, and let $waitT := Ldown$

Set $dueT := cT + waitT$, and flip the state of the span by $f_{\Pr ev} := 1 - f_{\Pr ev}$.

If $dueT \geq Te$, let

$wT := Te$,

$remT := dueT - Te$

If $dueT < Te$, set

$wT := dueT$, $remT := 0$.

(3) Case 2: Ph is an inactive phase ($f_{Lt} = 1$).

If $f_{\Pr ev} = 1$, meaning the previous span is in up state. In this case, as Ph is inactive phase, the component will keep in up state until the end of Ph. Therefore, let $wT := Te$.

If $f_{\Pr ev} = 0$, meaning the previous span is in down state, do as follows.

If $remT = 0$, set

$dueT := Te$

$f_{\Pr ev} = 1$ (flip the span state)

If $remT > 0$, then

$dueT := cT + remT$

If $dueT \geq Te$, let

$wT := Te$,

$remT := dueT - Te$

If $dueT < Te$, set

$wT := dueT$, $remT := 0$.

Step 3: Update the simulation clock to the next possible state change time, namely, $cT := wT$

It should be noted that the there is no restriction on the types of failure and repair time distributions. We just need to sample from the corresponding distributions in the proposed simulation procedure.

3 EXAMPLE

3.1 System Description

To illustrate our approach, we use a simplified example of a TT&C mission as shown in Fig.3. This system is composed of 5 components, and its mission has 3 phases. Each phase corresponds to a time window for the ground facility to provide TT&C service to a spacecraft according the schedule of TT&C resources prior to mission implementation.

The scenario of the mission and the requirements for mission success are described as follows. In phase 1, it is required to monitor the status of the spacecraft continuously during its pass over, and the TT&C system need to remain operational without any failure throughout the phase. In phase 2, the system needs to send instructions upward to another spacecraft for remote control; this job requires components c_1, c_4 to work jointly, and the system must work normally for a time interval of no less than T_d. So, this phase is a time redundancy phase. Similarly to phase 1, phase 3 is an ordinary phase in which the system needs to be operational throughout the phase.

Table 1 shows the structure functions and time durations (in minutes) of each phase, where ϕ_i denotes the structure function of phase i, and x_i is the indicator variable for component c_i. All the components has exponential failure and repair time distributions. Table 2 gives the failure and repair rates per minute of each component of the system. Here, we suppose $T_d = 6$ mins.

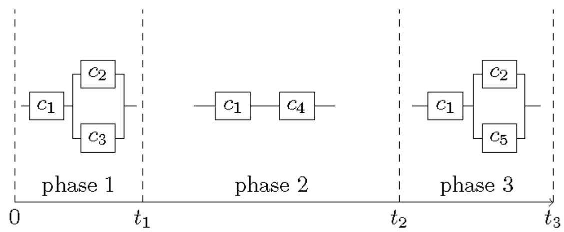

Figure 3 An example PMS

For this system, during phase 2, component c_2 is inactive, which means that c_2 can be repaired, but will not fail if it is up at the start time of the phase, or after being repaired during the phase.

To be more specific, the criteria for mission success are given as follows:1) the mission succeeds if the mission succeeds in all three phases. 2) the mission succeeds in phases 1 and 3 only if the system is operational throughout all the phase time. 3) the mission success in phase 2 requires that the system is operational for a minimum time length of T_d.

Table 1 Structure Functions and Durations

phase	structure function	duration
phase 1	$x_1(x_2 + x_3)$	10
phase 2	$x_1 x_4$	20
phase 3	$x_1(x_2 + x_3)$	12

Table 2 Distribution Parameters

Component	failure rate	repair rate
c_1	1/400	1/8
c_2	1/150	1/10
c_3	1/120	1/10
c_4	1/170	1/10
c_5	1/200	1/10

3.2 Results of Simulation

The proposed simulation procedure has been implemented by code on Macbook Air with an Intel 1.3GHz processor and 4GB of memory.

Table 3 presents the point estimates R as well as the 99% confidence intervals (CI) of the mission reliability obtained by different numbers of distinct simulation runs.

In Table 4, we give some selected values of mission reliability with different values of T_d, when the number of simulation runs is 350000, where R are the point estimated mission reliabilities of the system.

Table 3 Point and Confidence Interval Estimations

Number of Runs	R	CI
20000	0.9104	[0.9052, 0.9157]
50000	0.9068	[0.9035, 0.9102]
100000	0.9080	[0.9057, 0.9104]
150000	0.9081	[0.9062, 0.9100]
200000	0.9077	[0.9061,0.9094]
250000	0.9081	[0.9067,0.9096]
300000	0.9082	[0.9069,0.9096]
350000	0.9082	[0.9069, 0.9094]

Table 4 Reliability with different T_d

T_d	0	6	15	20
R	0.922	0.908	0.827	0.794

From the table we can see that the mission reliability will increase when T_d decreases. When T_d approaches the length of the second phase duration 20, it can be expected the resulting mission reliability will approach the estimate of the mission reliability of the PMS when it has no time redundancy, which can be calculated using an ordinary CTMC model as 0.79468, instead of 0.908 when time redundancy is considered as in Table 4. Therefore, we can see that the value of mission reliability would be seriously under estimated if the time redundancy is not considered.

6 CONCLUSIONS

Some PMSs have time redundancy in phases during mission execution. This paper defines mission reliability for these PMS, and presents a Monte-Carlo simulation approach for estimating the mission reliability of such PMS. Both point and confidence interval estimations are provided. Although the approach is illustrated by using a PMS with components of exponential distributions, it can also be applied directly to cases with nonexponential distributions. This is one of the main advantages of the simulation approach..

Further research work following on from this paper includes: developing a graphical interface for specifying the configuration of the PMS, its structure functions and related probability distributions of its components; extending the simulation approach for mission reliability of type II as defined in Ref. [16].

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